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The Shortest Time and/or the Shortest Path Strategies in a CA FF Pedestrian Dynamics Model

Ekaterina S.Kirik*
Institute of Computational Modelling,
Siberian Branch of Russian Academy of Sciences,
Akademgorodok 50/44, Krasnoyarsk, 660036
Russia
Tat'yana B.Yurgel'yan†
Institute of Mathematics,
Siberian Federal University,
Svobodny 79, Krasnoyarsk, 660041
Russia
Dmitriy V.Krouglov‡
V.N. Sukachev Institute of Forest,
Siberian Branch of Russian Academy of Sciences,
Akademgorodok 50/28, Krasnoyarsk, 660041
Russia

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The paper deals with a mathematical model of a pedestrian movement based on a stochastic cellular automata (CA) approach. A basis of the model obtained is the Floor Field (FF) model. FF models imply that virtual people follow the shortest path strategy. However, in reality people follow the strategy of the shortest time as well. The focus of the paper is on mathematical formalization and implementation of these features into a model of pedestrian movement. Some results of computer simulations are presented.

Keywords: cellular automata, pedestrian dynamics, transition probabilities, artificial intelligence.

Introduction

A stochastic cellular automata (CA) model of pedestrian flow is considered here. Our model stems from the stochastic floor field (FF) CA model [1] that provides pedestrians with a map which "shows" the shortest distance from the current position to the destination. While moving people follow at least two strategies — the shortest path and the shortest time. Strategies may vary, cooperate, and compete depending on the current position. In this paper we focus on mathematical formalization and implementation into the model these behavioral aspects of the decision making process.

This paper is a next attempt [2] to extend the basic FF model towards a behavioral aspect making more flexible/realistic decision making process and improve simulation of individual and collective dynamics of people flow.

*e-mail address: kirik@icm.krasn.ru
†e-mail address: tanthik@krasu.ru
‡e-mail address: volgurk@mail.ru

1. Statement of the problem

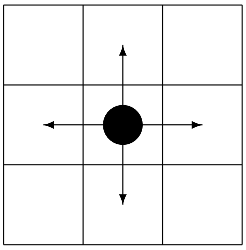
The space (plane) is known and sampled into cells $40cm \times 40cm$ which can either be empty or occupied by one pedestrian (particle) only [1]. Cells may be occupied by walls and other fixed obstacles. So the space is presented by 2 matrices:

$$f_{ij} = \begin{cases} 1, & \text{cell } (i,j) \text{ is occupied by a pedestrian;} \\ 0, & \text{cell } (i,j) \text{ is empty,} \end{cases}$$

$$w_{ij} = \begin{cases} 1, & \text{cell } (i,j) \text{ is occupied by an obstacle;} \\ 0, & \text{cell } (i,j) \text{ is empty.} \end{cases}$$

A *Static Floor Field* (SFF) S is used in the model. The field S coincides with the sampled space. The value of each $S_{i,j}$ is the shortest distance from the cell (i,j) to the nearest exit; i.e., S increases radially from the exit cells where $S_{i,j}$ are zero. It doesn't evolve with time and isn't changed by the presence of the particles. One can consider S as a map that pedestrians use to move to the nearest exit.

The initial positions of the people are known. The destination for each pedestrian is the nearest exit. Each particle can move to one of the four its adjacent cells or to stay in present cell (the von Neumann neighborhood) at each discrete time step $t \to t+1$ — fig. 1; i.e., $v_{max} = 1$.



0	$p_{i-1,j}$	0
$p_{i,j-1}$	$p_{i,j}$	$p_{i,j+1}$
0	$p_{i+1,j}$	0

Fig. 1. Target cells for a pedestrian in the next time step [1]

Generally speaking, the direction for each particle at each time step is random and determined in accordance with the transition probabilities distribution (and transition rules).

Thus the main problem is to determine the "correct" transition probabilities (and transition rules).

2. Solution

2.1. Update rules

A typical scheme for stochastic CA models is used here. There is a step of some preliminary calculations. Then at each time step the transition probabilities are calculated, and the directions are chosen. If there are more then one candidate to one cell then a conflict resolution procedure is applied, and a simultaneous transition of all particles is made.

In our case the *preliminary step* includes calculations of SFF S. Each cell $S_{i,j}$ stores the shortest discreet distance to the nearest exit. The unit of such distance is a number of steps. To calculate the field S (and for this purpose only) we admit diagonal transitions and assume that a vertical and a horizontal movement to the nearest cell has a length of 1; the length of a diagonal movement to the nearest cell is $\sqrt{2}$. (It is clear that a movement through a corner of walls or columns is forbidden and only a roundabout movement is admitted in such cases.) These assumptions bring the discreet distance closer to the continuous one.

The probabilities of moving from the cell (i, j) to each of the four adjacent cells are calculated in the following way:

$$p_{i-1,j} = \frac{\tilde{p}_{i-1,j}}{Norm_{i,j}}, p_{i,j+1} = \frac{\tilde{p}_{i,j+1}}{Norm_{i,j}}, p_{i+1,j} = \frac{\tilde{p}_{i+1,j}}{Norm_{i,j}}, p_{i,j-1} = \frac{\tilde{p}_{i,j-1}}{Norm_{i,j}}, \quad (1)$$

where $Norm_{i,j} = \tilde{p}_{i-1,j} + \tilde{p}_{i,j+1} + \tilde{p}_{i+1,j} + \tilde{p}_{i,j-1}$.

Moreover

$$p_{i-1,j} = 0, \quad p_{i,j+1} = 0, \quad p_{i+1,j} = 0, \quad p_{i,j-1} = 0 \quad (2)$$

only if

$$w_{i-1,j} = 1, \quad w_{i,j+1} = 1, \quad w_{i+1,j} = 1, \quad w_{i,j-1} = 1 \quad (3)$$

correspondingly.

The probability of keeping the current position is not directly calculated. However, the decision rules allow this possibility modeling the situation when a person needs to wait before moving.

The decisions rules are the following [2]:

1. If $Norm_{i,j} = 0$ then motion is forbidden, otherwise the target cell $(l, m)^*$ is chosen randomly using the transition probabilities.

2. (a) If $Norm_{i,j} \neq 0$ and $(1 - f^*_{l,m}) = 1$ then the target cell $(l, m)^*$ is fixed.
   (b) If $Norm_{i,j} \neq 0$ and $(1 - f^*_{l,m}) = 0$ then the cell $(l, m)^*$ is not available for moving and a "people patience" can be realized. To do it probabilities of the cell $(l, m)^*$ and all other occupied the nearest neighbors are given to an opportunity not to leave the present position. A target cell is randomly chosen again among empty neighbors and the present position.

3. Whenever two or more pedestrians have the same target cell, the movement of all involved pedestrians is denied with the probability μ; i.e. all pedestrians remain at their places [1]. One of the candidates moves to the desired cell with the probability $1 - \mu$. A pedestrian that is allowed to move is chosen randomly.

4. Pedestrians that are allowed to move perform their motion to the target cell.

5. Pedestrians that stand in exit cells are removed from the room.

These rules are applied to all particles at the same time; i.e., parallel update is used.

2.2. How to calculate probability?

The main focus in this paper is on transition probabilities. In normal situations people carefully choose their route (see [3] and reference therein). Pedestrians keep certain distance from other people and obstacles. The more hurried a pedestrian is and the more tight the crowd is the smaller this distance is. While moving people follow at least two strategies — the shortest path and the shortest time.

In FF models people move to the nearest exit, and their wish to move there doesn't depend on the current distance to the exit. From the probabilistic point of view this means that for each particle among all the nearest neighbor cells a neighbor with the smallest S should have the largest probability. So the main driving force for each pedestrian is to minimize SFF S at each time step. But in this case only a strategy of the shortest path is mainly realized, and a slight regard to an avoidance of congestions is supposed. This is not realistic for people movement.

The idea to improve the dynamics in a FF model is to introduce an environment analyzer in a probability formula. It should decrease the influence of the short path strategy and increase the possibility to move in a direction with favorable conditions for moving. This will provide some kind of "trade off" between two main strategies.

In this paper we introduce a revised idea of the environment analyzer [2] and make an attempt to mathematically formalize a complex decision making process that people do choosing their path — while moving their strategies may vary: cooperate, coincide, and compete depending on the current position and environment; i.e., depending on the place and time.

At first let us present a probability formula and later we will discuss it in details. For example,the transition probability to move from a cell (i, j) to the upper neighbor is:

$$p_{i-1,j} = Norm_{i,j}^{-1} A_{i-1,j}^{SFF} A_{i-1,j}^{people} A_{i-1,j}^{wall} (1 - w_{i-1,j}). \tag{4}$$

Here

- $A_{i-1,j}^{SFF} = \exp\left(k_S \triangle S_{i-1,j}\right)$ — the main driven force:

  1. $\triangle S_{i-1,j} = S_{i,j} - S_{i-1,j}$;
  2. $k_S \geqslant 0$ — a sensitivity parameter (model parameter) that can be interpreted as the knowledge of the shortest way to the destination point, or as a wish to move to the destination point. $k_S = 0$ means that pedestrians don't use information from the SFF S and move randomly. The higher k_S is the more directed is the movement of pedestrians.

  As far as SFF depict direct distance from each cell to the nearest exit then $\triangle S_{i-1,j} > 0$ if cell $(i-1, j)$ is closer to exit than the current cell (i, j). $\triangle S_{i-1,j} < 0$ if the current cell is closer. And $\triangle S_{i-1,j} = 0$ if the cells (i, j) and $(i-1, j)$ are equidistant to the exit.

  In contrast with other authors that deal with the FF model (e.g., [1, 4, 5, 6]) and use pure values of the field S in the probability formula we propose to use $\triangle S_{i-1,j}$ only. From the mathematical point of view it is the same but computationally this trick has a great advantage. The values of SFF may be too large (it depends on the size of the space), and $\exp\left(k_S S_{i-1,j}\right)$ is uncomputable. This is a significant restriction of that models. At the same time $0 \leqslant \triangle S_{i-1,j} \leqslant 1$, and problem of computing $A_{i-1,j}^{SFF}$ is absent;

- $A_{i-1,j}^{people} = \exp\left(-k_P D_{i-1,j}(r_{i-1,j}^{*})\right)$ — a factor that takes into account a people density in the direction:

  1. $r_{i-1,j}^{\star}$ — the distance to the nearest obstacle in this direction ($r_{i-1,j}^{\star} \leqslant r$);
  2. $r > 0$ — the "visibility" radius (a model parameter) which is the maximal distance (number of cells) at which the pedestrian can look through to collect information about the density and possible obstacles;

3. density $0 \leqslant D_{i-1,j}(r^*_{i-1,j}) \leqslant 1$; if all $r^*_{i-1,j}$ cells are empty in this direction then $D_{i-1,j}(r^*_{i-1,j}) = 0$; if all $r^*_{i-1,j}$ cells are occupied by people in this direction then $D_{i-1,j}(r^*_{i-1,j}) = 1$. We estimate density by using idea of the kernel Rosenblat-Parzen's [7] density estimate, and

$$D_{i-1,j}(r^*_{i-1,j}) = \frac{\sum\limits_{m=1}^{r^*_{i-1,j}} \Phi\left(\frac{m}{C(r^*_{i-1,j})}\right) f_{i-m,j}}{r^*_{i-1,j}},$$

were

$$\Phi(z) = \begin{cases} \left(0.335 - 0.067(z)^2\right) 4.4742, & |z| \leqslant \sqrt{5}; \\ 0; & |z| > \sqrt{5}, \end{cases} \qquad (5)$$

$C(r^*_{i-1,j}) = \dfrac{r^*_{i-1,j} + 1}{\sqrt{5}}$;

4. $k_P \geqslant k_S$ — a people sensitivity parameter (a model parameter) determines the influence of the people density. The higher k_P is the more pronounced the strategy of the shortest path is.

- $A^{wall}_{i-1,j} = \exp\left(-k_W(1 - \frac{r^*_{i-1,j}}{r})\tilde{1}(\triangle S_{i-1,j} - \max \triangle S_{i,j})\right)$ — a factor that takes into account walls and obstacles:

  1. $k_W \geqslant k_S$ — a wall sensitivity parameter (a model parameter) determines the influence of walls and obstacles;
  2. $\max \triangle S_{i,j} = \max\{\triangle S_{i-1,j}, \triangle S_{i,j+1}, \triangle S_{i+1,j}, \triangle S_{i,j-1}\}$,
     $\tilde{1}(\phi) = \begin{cases} 0, & \phi < 0, \\ 1 & \text{otherwise.} \end{cases}$
     The idea of the function $\tilde{1}(\triangle S_{i-1,j} - \max \triangle S_{i,j})$ comes from a the fact that people avoid obstacles only moving towards a destination point. But if people take detours (that means not minimizing the SFF) approaching obstacles is not avoiding.

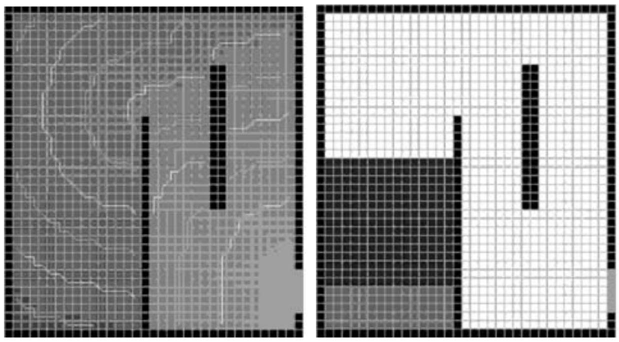
- NOTE that only walls and obstacles turn the probability to "zero".

The probabilities to move from a cell (i,j) to each of the four neighbors are:

$$p_{i-1,j} = Norm^{-1}_{i,j} \exp\big[k_S \triangle S_{i-1,j} - k_P D_{i-1,j}(r^*_{i-1,j}) - \\ - k_W(1 - \frac{r^*_{i-1,j}}{r})\tilde{1}(\triangle S_{i-1,j} - \max \triangle S_{i,j})\big](1 - w_{i-1,j}); \quad (6)$$

$$p_{i,j+1} = Norm^{-1}_{i,j} \exp\big[k_S \triangle S_{i,j+1} - k_P D_{i,j+1}(r^*_{i,j+1}) - \\ - k_W(1 - \frac{r^*_{i,j+1}}{r})\tilde{1}(\triangle S_{i,j+1} - \max \triangle S_{i,j})\big](1 - w_{i,j+1}); \quad (7)$$

$$p_{i+1,j} = Norm^{-1}_{i,j} \exp\big[k_S \triangle S_{i+1,j} - k_P D_{i+1,j}(r^*_{i+1,j}) - \\ - k_W(1 - \frac{r^*_{i+1,j}}{r})\tilde{1}(\triangle S_{i+1,j} - \max \triangle S_{i,j})\big](1 - w_{i+1,j}); \quad (8)$$



a) Field S b) Initial positions

Fig. 2.

$$p_{i,j-1} = Norm_{i,j}^{-1} \exp\big[k_S \triangle S_{i,j-1} - k_P D_{i,j-1}(r^*_{i,j-1}) - \\ - k_W (1 - \frac{r^*_{i,j-1}}{r}) \tilde{1}(\triangle S_{i,j-1} - \max \triangle S_{i,j})\big](1 - w_{i,j-1}). \quad (9)$$

In (6)-(9) the product $A^{people}A^{wall}$ is the environment analyzer that deals with people and walls. The parameters k_P and k_W allow one to tune sensitivity of the model to the people density and the approaching to obstacles correspondingly. As far as $0 \leqslant \triangle S \leqslant 1$, $0 \leq D(r^*) \leqslant 1$, and $0 \leqslant 1 - \dfrac{r^*}{r} \leqslant 1$ both parameters shouldn't be less then k_S. The term A^{wall} is only to avoid obstacles ahead; we will not discuss it here, and let $k_W = k_S$

To follow the shortest path strategy means to take detours around high density regions if it is possible. The term A^{people} works as a reduction of the main driving force (that provides the shortest path strategy), and the probability of detours becomes higher. The higher $k_P \geqslant k_S$ is the more pronounced the shortest time strategy is. Note that the low people density makes influence of A^{people} small, and the probability of the shortest path strategy increases for the particle.

3. Simulations

Here we present some simulation results to demonstrate that our idea works. We use one space and compare two sets of parameters. The size of space is $14.8m \times 13.2m$ (37 cells $\times$ 33 cells) with one exit ($2.0m$). Recall that the space is sampled into cells of size $40cm \times 40cm$ which can either be empty or occupied by one pedestrian only. The static field S is presented in fig. 2a. Fig. 2b shows the starting positions of particles. They move towards the exit with $v = v_{max} = 1$.

Here we don't present some quantity results and only demonstrate a quality difference of the flow dynamics for two sets of model parameters for the model presented.

The first set of parameters is $k_S = k_W = 4$, $k_P = 6$, $r = 10$. The second set is $k_S = k_W = 4$, $k_P = 18$, $r = 10$. The following moving condition are reproduced by both sets — pedestrians know a way to the exit very well; they want go to the exit (it is determined by k_S); a visibility is good (r); attitude to walls is "loyal" ($k_W = k_S$). The only parameter that varies here is k_P.

In the first case ($k_P = 6$) a prevailing moving strategy is the shortest path. Fig. 3 presents an evacuation in different moments for this case.

The other set of parameters $k_S = k_W = 4$, $k_P = 18$, $r = 10$ (see fig. 4) allows to realize both strategies depending on conditions. Recall that the term A^{people} in (6)-(9) only works if the

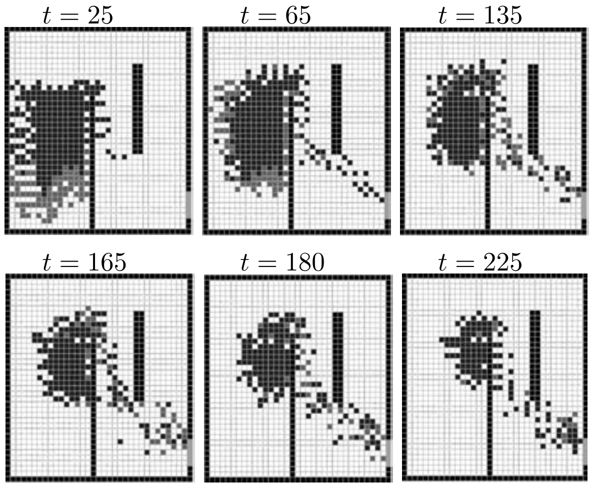


Fig. 3. Evacuation for 300 people, $k_S = k_W = 4$, $r = 10$, $k_P = 6$

people density $D(r^\star) > 0$, and it reduces the probability of the shortest path strategy depending on the density.

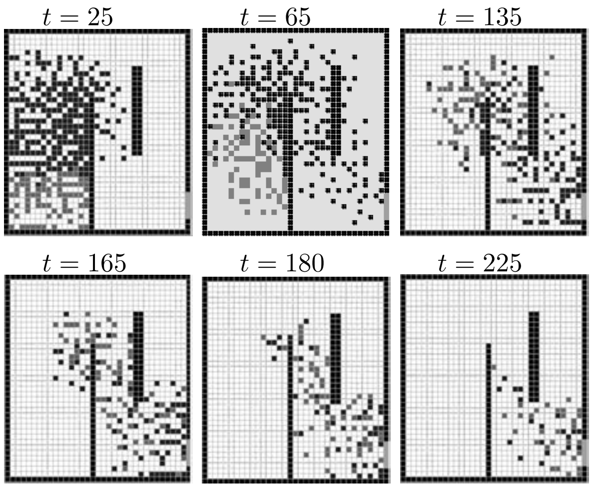


Fig. 4. Evacuation for 300 people, $k_S = k_W = 4$, $r = 10$, $k_P = 18$

Conclusion

Fig. 3-4 show a great difference in the flow dynamics that obtained by following only one movement strategy and by "keeping in mind" both strategies at a time. The case of $k_P = 18$, i.e., when both strategies of the shortest path and the shortest time are well pronounced, gives a more realistic shape of flow. A model dynamics needs a careful investigation and this will be the subject of future research. The necessity of a spatial adaptation of k_P is already clear.

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